CALCULATIONS FOR A CYLINDRICAL ELECTRIC ARC WITH ALLOWANCE FOR ENERGY TRANSFER BY RADIATION WITH THE HYDROGEN AT A PRESSURE OF 100 ATM

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An approximate method is described for the consideration of energy transfer by radiation during the utilization of real properties of a gas (in particular, the frequency-dependent absorption coefficient under conditions of local thermal equilibrium). With increasing pressure, it becomes necessary to take self-absorption into account over almost the entire frequency spectrum.

Calculations are carried out for a wall-stabilized cylindrical electric arc in hydrogen as an example for a pressure of 100 atm and channel radii of 0.3, 1, and 3 cm at values of current strength up to the order of 10^4 A. The strong effect of radiation on the current-voltage characteristic of the arc, the gas temperature, and the nature of its distribution over the arc radius is demonstrated.

The process of energy transfer by radiation plays a significant and sometimes predominant role in the thermal balance of electric arcs with high current strengths [1–9]. Calculations have been performed for cylindrical arcs in atmospheres of argon and hydrogen [5, 7] with allowance for energy transfer by radiation and for atmospheric pressure in which case the gas is essentially transparent to radiation. Approximate estimates were obtained for the self-absorbed portion of the radiation.

The role played by radiation increases with increasing current strength, arc radius, and pressure, while self-absorption in this process extends over an increasingly large region of the spectrum. Hence, calculations must be carried out for the arc if conditions are such that the gas in the arc does not transmit radiation.

In [10–13], an approximate method was developed for taking into account energy transfer by radiation in the presence of intense self-absorption as applied to heat transfer problems under conditions of local thermal equilibrium with allowance for the variation of the absorption coefficient as a function of the frequency. The conditions for local thermal equilibrium in an arc passing through an argon or hydrogen atmosphere are fulfilled for pressures greater than atmospheric pressure and for current strengths greater than ~10 A [14–16]. The results of [10–12] were used as the foundation for calculations based on an electric arc in argon at atmospheric pressure, under which conditions, self-absorption affects only the transitions to the ground state. The part played by radiation in the heat transfer process is smaller than the part played in the energy transfer by conduction. Calculations confirmed the results of [5, 7].

The role of energy transfer by radiation in the energy balance of the arc increases with increasing pressure, while in turn, the role of the continuous spectrum increases for the radiation. The results of calculations performed for a wall-stabilized arc burning in an atmosphere of hydrogen at a pressure of 100 atm are given in the present paper. In this case, almost the entire energy supply is lost by radiation. The approximate method of accounting for energy transfer by radiation is demonstrated by an example.

NOTATION

ρ and T are the gas density and temperature, respectively; u is the velocity; c_p is the heat capacity of the gas at constant pressure; $\varkappa$ is the coefficient of thermal conductivity; σ is the coefficient of electrical conductivity; x and r are the cylindrical coordinates; R_0 is the channel radius; I is the current strength; E is the electric field strength; $u_\nu{}^\circ$ is the equilibrium value of radiation energy density; u_ν is the value of radiation energy density; ν is the radiation frequency; φ is the divergence of energy flux density transported by radiation; k_ν is the absorption coefficient; c is the speed of light; ε_i is the emissivity of the i-th region of the spectrum.

Averaging symbols: $\langle k' \rangle_i$ is the mean value of the absorption coefficient in the i-th region of the spectrum, averaged on the basis of Planck's law; $\langle k'' \rangle_i$ is the same, averaged on the basis of Rosseland's law,

$$\langle k' \rangle = \int k_\nu u_\nu{}^\circ d\nu \Big/ \int u_\nu{}^\circ d\nu ;$$

$$\langle k'' \rangle = \int \frac{du_\nu{}^\circ}{dT} d\nu \Big/ \int \frac{1}{k_\nu} \frac{du_\nu{}^\circ}{dT} d\nu ;$$

$$\varepsilon_i = u_i{}^\circ \Big/ \int_0^\infty u_\nu{}^\circ d\nu ; \quad u_i{}^\circ = \int_{\Delta\nu_i} u_\nu{}^\circ d\nu ; \; u_i = \int_{\Delta\nu_i} u_\nu d\nu ;$$

$$\langle k \rangle = (\langle k' \rangle \langle k'' \rangle)^{1/2} .$$

1. Calculations are performed for a cylindrical arc with longitudinally varying characteristics. The value of the electric field strength E is constant across and along the axis of the arc. Thermal local equilibrium is postulated.

The heat conduction equation

$$\frac{\partial T}{\partial x'} = \frac{1}{r} \frac{\partial}{\partial r} \left(\varkappa r \frac{\partial T}{\partial r} \right) + \sigma E^2 - \varphi \tag{1.1}$$

is solved under the following conditions:

$$r = 0; \partial T/\partial r = 0; r = R_0; \; T = T_1 = 300^\circ \mathrm{K},$$

$$x' = 0; \; T(0, r) = F(r), \tag{1.2}$$

for given channel radius R_0 and current strength I. Here F(r) is the given temperature distribution at the inlet section, and x' the value of the x coordinate divided by the quantity $\rho u c_p$ = const, which is permissible, since, in the case under consideration, the hydrodynamic picture has no influence on the results. The value of the electric field strength is

$$E = I \left(2\pi \int_0^{R_0} \sigma r dr \right)^{-1} .$$

Equation (1.1) was solved with the aid of a finite-difference scheme. Since the temperature profile at the channel wall experiences abrupt variations, a variable which extends the region adjacent to the wall was substituted for the radius.

Calculations were performed prior to the onset of steady-state conditions, the transient state being evaluated on the basis of the variations of E, although the temperature profile at the wall was still not completely steady.

The composition of the gas was determined on the basis of chemical equilibrium formulas, with allowance for the decrease in ionization potential. The coefficient of electrical conductivity σ and the coefficient of thermal conductivity $\varkappa$ were calculated (with allowance for ionization-energy transfer) from relations in elementary kinetic theory. The relations of σ (sec^{-1}, curve 1) and $\varkappa$ (erg · cm^{-1} · degree^{-1}, curve 2) are given in Fig. 1.

2. The divergence of the energy current density transported by radiation φ was calculated with allowance for the actual dependence of the absorption coefficient on frequency.

The entire spectral region was divided into subregions and in each, the absorption coefficient was averaged by a method proposed in [10–13]. The number of regions into which the spectrum had to be divided proved to be from a practical standpoint, feasible. In the case of low gas densities (atmospheric pressure), the transitions to the ground state and to the first excitation state are usually self-absorbed. The remaining portions of the spectrum are transparent. Spectral lines with the same temperature dependence of the absorption coefficient can be combined into groups [10–11]. In the case of high gas densities (high pressures), almost the entire spectrum is self-absorbed. Due to the substantial decrease in the ionization potential, however, nearly the entire discrete spectrum becomes continuous while the remaining spectral lines are insignificant in view of the strong continuous spectrum.

The radiation-transport equation is replaced by an approximate system of equations obtained by the method of spherical harmonics. In our paper, use was made of the P_1 approximation (diffusion approximation), the accuracy of which was verified in [10–13].

In the diffusion approximation, the quantity φ which appears in the equation is determined from the formula

$$\varphi = \sum_i \varphi_i = c \sum_i \langle k \rangle_i (u_i^\circ - u_i) \cdot \qquad (2.1)$$

Summation is performed over all the $\Delta \nu_i$ regions into which the spectrum is subdivided. For each region, the quantity u_i is determined from the equation [17]

$$-\frac{1}{3\langle k \rangle_i} \frac{1}{r} \frac{d}{dr}\left[\frac{r}{\langle k \rangle_i} \frac{du_i}{dr}\right] = u_i^\circ - u_i\,; \qquad (2.2)$$

with the boundary conditions

$$r = 0;\quad du_i/dr = 0\,;$$
$$r = R_0;\quad -\frac{1}{3\langle k \rangle_i} \frac{du_i}{dr} = \frac{u_i}{2}\,. \qquad (2.3)$$

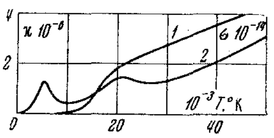

Fig. 1

This equation was solved with the aid of a finite-difference scheme, using a pivot method for each frequency range.

At a pressure of 100 atm and temperatures higher than 10 000° K, emission in hydrogen experiences strong self-absorption for almost the entire spectrum. In order to calculate energy transport by radiation at temperatures of several tens of thousands of degrees, one must take into account the entire spectral region of importance with respect to energy. Consideration was given to the frequency range from $2 \cdot 10^{14}$ sec^{-1} to $5.4 \cdot 10^{15}$ sec^{-1}. The large decrease in the ionization potential by as much as 2.5 eV, leads to conditions in practice under which the first and second excitation levels occur.

Changes in the photoionization cross section from the ground state and the excitation levels are taken into account on the basis of recommendations in [18]. The computations are simplified by extending the cross sections of the respective processes into the long-wave region of the spectrum by a maximum value that corresponds to a temperature of 20 000° K, and referring to this cross section for any temperature. This approximates the role of the spectral lines near the threshold frequencies. The simplifications introduced mean that the first line in the Lyman L_α series is the only one that is retained among the discrete transitions. The contour of these lines was calculated in several papers by Kolb, Grimm, and Schon. A survey of their work and the contours of the hydrogen lines is given in [19].

Accepting a permissible error of several per cent, the region of the spectrum under consideration was divided into 1, . . . , 6 regions (Fig. 7). The regions 1, 2, and 3 extend from $2 \cdot 10^{14}$ to $2.2 \cdot 10^{15}$ sec^{-1}, the spectral region associated with recombination at various levels of excitation with free-free transitions in proton and negative-ion fields, and with photodetachment from negative ions. The absorption coefficient is small in these processes at low temperatures, so that radiation which is intensely self-absorbed in the center of the arc experiences little blocking in the peripheral layers of the cold gas.

Regions 4 and 5 extend over the portion of the spectrum that corresponds to the L_α line. Two identical regions measuring $7 \cdot 10^{13}$ sec^{-1} each are located on either side of the center line segment which has a width of 4×10^{13} sec^{-1} and which has been neglected because of its small contribution to the energy flux as a result of its extremely high optical density. Due to the symmetry of the line, the two identical regions are incorporated into the same region of the spectrum – region 5. The same applies to the edge regions of the line, which have a width of 1.5×10^{14} sec^{-1} each and are incorporated into the spectral region 4.

Region 6 incorporates the spectrum associated with photoionization from the ground state.

The values of the absorption coefficient in all the spectral regions 1, . . . , 6 were averaged with the aid of formula

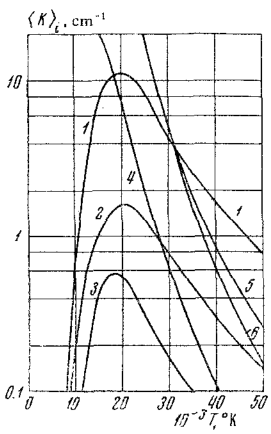

Fig. 2

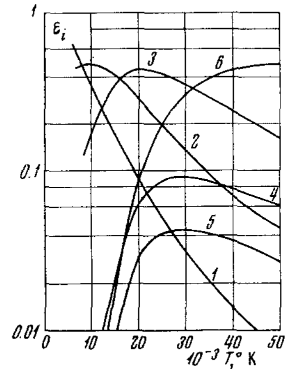

Fig. 3

$$\langle k \rangle = (\langle k' \rangle \ \langle k'' \rangle)^{1/2}$$

The values of $\langle k \rangle_i$ obtained are given in Fig. 2. The numbers of the curves denote the numbers of the spectral regions. The values of $\langle k \rangle_i$ for the regions 4–6 decrease monotonically with increasing temperature. Within these spectral regions, the radiation from the center of the arc is self-absorbed in the cold layers of the gas. Figure 3 shows the temperature dependent behavior of the emissivity ε_i for the corresponding regions of the spectrum.

3. Results of computations. Computations were performed for channel radii of 0.3, 1, and 3 cm for various current strengths. Figure 4 shows the generalized current-voltage characteristics $ER_0 = f(I/R_0)$. The dashed curve corresponds to calculations performed without allowance for energy transport by radiation, curve 1 to calculations performed for a channel radius R_0 = 0.3 cm, curve 2 to calculations for R_0 = 1 cm, and curve 3 to calculations for R_0 = 3 cm. The circles on the characteristic curves indicate the temperatures at the axis of the arc: curve 1 (from left to right)—12 000, 14 000, 16 000, and 20 000° K; curve 2—11 000, 14 000, and 18 000° K; curve 3—10 000, 14 000, and 18 000° K.

Figure 5 shows the temperature profiles in the arc for the corresponding values of the channel radius. The numbers at the curves indicate the current strength in amperes.

As an example, Fig. 6 shows the behavior of the divergence of the energy flux density transported by radiation φ, as a function of distance from the arc axis for R_0 = 0.3 cm, and at various current strengths. The behavior of φ_i in each region of the spectrum is of definite interest. Values of φ_i for all frequency ranges are given with a common scale in Fig. 7 for F_0 = 0.3 cm and I = 3000 A. The figure also shows the temperature profile and a schematic representation of the behavior of the absorption coefficient as a function of frequency.

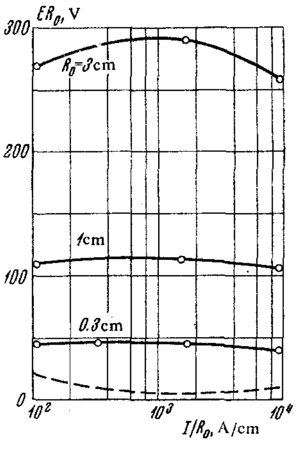

Fig. 4

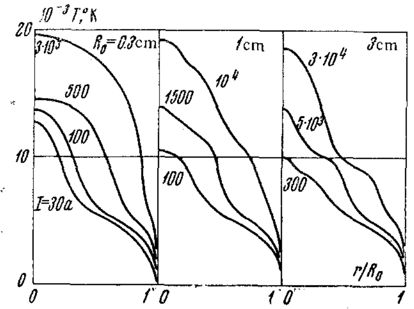

Fig. 5

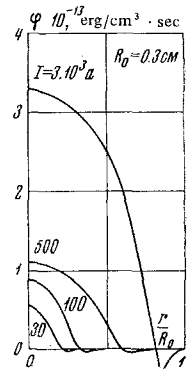

Fig. 6

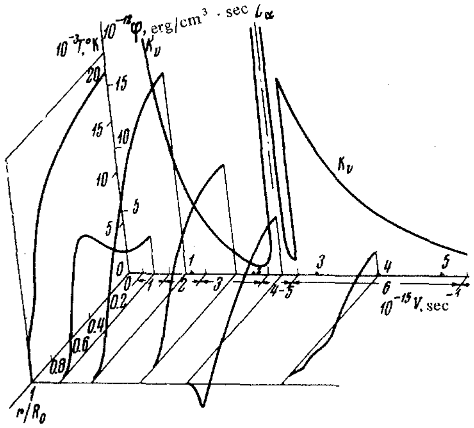

Fig. 7

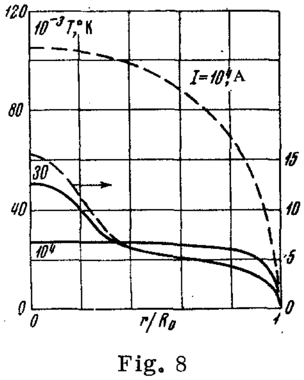

Fig. 8

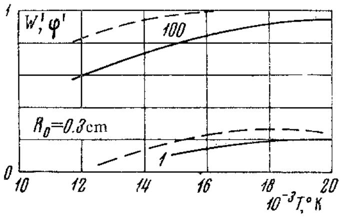

Fig. 9

The significance of energy transport by radiation in a thermal balance of the arc and in the formation of a temperature profile may be determined on the basis of data given in Fig. 8. For $R_0 = 0.3$ cm and a current strength of 30 A (temperature scale on the right side), or a current strength of 10 000 A (temperature strength on the left side) the figure shows two different temperature profiles. The temperature profiles are calculated with (solid curves) and without (dashed curves) allowance for radiation.

For I = 30 A, a slight temperature drop at the axis produces a relatively small change in the temperature profile, whereas for I = = 10 000 A, the changes in the temperature profile are quite substantial. For $R_0 = 0.3$ cm, the solid curve in Fig. 9 shows the value of the ratio of the power lost by radiation W per unit length of the arc to the input power IE: $W' = W/IE$. The dashed curve in the figure shows the value of $\varphi' = \varphi_0/\sigma_0 E^2$, which characterizes the role of radiation at the axis, as a function of the gas temperature at the axis of the arc; φ_0 and σ_0 are values at the channel axis. Even for $R_0 = 0.3$ cm, the major portion of the energy is transported by radiation. The role of radiation is still greater for larger channel radii. The numbers at the curves denote the value of the pressure in atmospheres. The results of computations for a pressure of one atmosphere are taken from [5].

The described method of calculating an arc with allowance for energy transport by radiation can be used for calculatons with arcs in atmospheres of other gases. The authors are indebted to L. M. Vetlutskaya and V. N. Vetlutskii for their assistance in the computations.

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